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RADIATION FROM A MOVING MAGNETON

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1. The rate of radiation of energy from a ring of electrons revolving in a circular orbit and from various other distributions of moving electric charges and magnetic poles has been calculated by G. A. Schott,[1] who finds that the rate of radiation of energy is almost invariably positive. This is certainly true in the case of a single electric pole describing a circular orbit as is indicated by the well known formulae of Larmor and Liénard for the rate of radiation. Thus electromagnetic theory in its present form lends no support to Bohr's idea of non-radiating orbits.

A steady distribution such as a Parson magneton which consists of a complete ring of electric charges following one another round the ring at a constant speed will evidently give no radiation when the ring is stationary as a whole, but as Schott remarks the ring may be expected to radiate energy when its centre has an acceleration.

Schott's results are so important that it is desirable that they should be confirmed by an independent method and an attempt has been made to devise a method by which the rate of radiation from a moving electric pole and magnetic doublet may be readily calculated. In two important cases we have confirmed Schott's surmise that the rate of radiation is positive.

2. Starting with the case of an *electromagnetic doublet*, i.e. an electric doublet and magnetic doublet which move together, we determine the electric force E and the magnetic force H from the equations

$$M \equiv H + iE = i \operatorname{rot} L$$

$$L = \frac{\partial G}{\partial t} + ic \operatorname{rot} G, \qquad G = \frac{1}{\nu} g(\tau)$$

$$4\pi g = q + ip - \frac{1}{c}(v \times p) + \frac{i}{c}(v \times q)$$

$$\nu = \xi'(\tau)[x - \xi(\tau)] + \eta'(\tau)[y - \eta(\tau)] + \zeta'(\tau)[z - \zeta(\tau)] - c^2(t-\tau)$$

$$\equiv r[(v \cdot s) - c]$$

$$r^2 = [x - \xi(\tau)]^2 + [y - \eta(\tau)]^2 + [z - \zeta(\tau)]^2 = c^2(t-\tau)^2. \qquad t \geq \tau.$$

In these equations v denotes the velocity at time τ of the moving point P whose coordinates at this instant are $\xi(\tau)$, $\eta(\tau)$, $\zeta(\tau)$; x, y, z are the coordinates of an arbitrary point Q; s is a unit vector in the direction of the line PQ; p and q are vectors representing the electric and magnetic moments at time τ; t is the time, and c the velocity of light.

From these equations we find that

$$E \times H = c^2 \frac{r^4}{\nu^4} s[g^* \cdot g_0^* - (s \cdot g^*)(s \cdot g_0^*) - i\{s \cdot (g^* \times g_0^*)\}]$$

where g^* is a certain complex vector and g^*_0 the conjugate complex vector. A similar expression may be obtained in the case of an electromagnetic pole and also in the case of the combination of an electromagnetic pole and an electromagnetic doublet. In the latter case the appropriate expression for g^* is

$$g^* = \frac{1}{\nu} b - 3\frac{r}{\nu^2}(s \cdot v')g' + \frac{3r^2}{\nu^3}(s \cdot v')^2 g - \frac{r}{\nu^2}(s \cdot v'')g$$

where

$$b = g'' + mv' + i\frac{m}{c}(v \times v'),$$

and

$$m = h + ie.$$

Here $4\pi e$ and $4\pi h$ are the electric and magnetic charges associated with the pole and primes denote differentiations with respect to τ.

Calculating the rate of radiation I across a very large sphere whose centre is at the moving point P we find that I may be represented by the real part of the following expression

$$\pi c^3 \left[\frac{8}{3} \frac{c^2 + 2v^2}{(c^2 - v^2)^4}(b \cdot b_0) - \frac{8}{(c^2 - v^2)^4}(v \cdot b)(v \cdot b_0) + \frac{16}{5} \frac{c^2 + v^2}{(c^2 - v^2)^5} \left\{ 12(v \cdot v')(b \cdot g_0') \right. \right.$$

$$\left. + 3v'^2(g' \cdot g_0') + 4(v \cdot v'')(b \cdot g_0) + 2v'^2(b \cdot g_0) + 2(v' \cdot v'')(g' \cdot g_0) + \frac{1}{3}v''^2(g \cdot g_0) \right\}$$

$$+ \frac{8}{5} \frac{3c^2 + 2v^2}{(c^2 - v^2)^6} \left\{ 24(v \cdot v')^2(g' \cdot g_0') + 16(v \cdot v')^2(b \cdot g_0) + \frac{144}{7} v'^2(v \cdot v')(g' \cdot g_0) \right.$$

$$+16(v\cdot v')(v\cdot v'')(g'\cdot g_0)+\frac{9}{7}v'^4(g\cdot g_0)+\frac{16}{7}(v\cdot v'')v'^2(g\cdot g_0)+\frac{32}{7}(v\cdot v')(v'\cdot v'')(g\cdot g_0)$$

$$+\frac{8}{3}(v\cdot v'')^2(g\cdot g_0)\Big\} + \frac{64}{7}\frac{2c^2+v^2}{(c^2-v^2)^7}\Big\{24(v\cdot v')^3(g'\cdot g_0) + 9v'^2(v\cdot v')^2(g\cdot g_0)$$

$$+8(v\cdot v'')(v\cdot v')^2(g\cdot g_0)\Big\} - \frac{8}{5}\frac{1}{(c^2-v^2)^4}\Big\{6(v\cdot b)(v'\cdot g_0') + 6(v\cdot g_0')(v'\cdot b)$$

$$+3(v'\cdot g')(v'\cdot g_0') + 2(v'\cdot b)(v'\cdot g_0) + (v'\cdot v'')(g'\cdot g_0) + (v''\cdot g')(v'\cdot g_0)$$

$$+(v'\cdot g')(v''\cdot g_0) + \frac{1}{3}(v''\cdot g)(v''\cdot g_0) + 2(v\cdot b)(v''\cdot g_0) + 2(v\cdot g_0)(v''\cdot b)\Big\}$$

$$-\frac{32}{5}\frac{1}{(c^2-v^2)^5}\Big\{12(v\cdot v')(v\cdot g_0')(v\cdot b)+6(v\cdot v')(v\cdot g')(v'\cdot g_0')+6(v\cdot v')(v'\cdot g')(v\cdot g_0')$$

$$+3v'^2(v\cdot g')(v\cdot g_0') + 2v'^2(v\cdot b)(v\cdot g_0) + 4(v\cdot v')(v\cdot b)(v'\cdot g_0) + 4(v\cdot v')(v'\cdot b)(v\cdot g_0)$$

$$+2(v'\cdot v'')(v\cdot g')(v\cdot g_0) + 2(v\cdot v')(v\cdot g')(v''\cdot g_0) + 2(v\cdot v'')(v\cdot g_0)(v'\cdot g')$$

$$+2(v\cdot v'')(v\cdot g')(v'\cdot g_0) + 2(v\cdot v')(v\cdot g_0)(v''\cdot g') + \frac{4}{3}(v\cdot v'')(v\cdot g)(v''\cdot g_0)$$

$$+\frac{1}{3}v''^2(v\cdot g)(v\cdot g_0)+\frac{18}{7}v'^2(v\cdot g')(v'\cdot g_0)+\frac{18}{7}v'^2(v\cdot g_0)(v'\cdot g')+\frac{4}{7}(v\cdot v'')(v'\cdot g)(v'\cdot g_0)$$

$$+\frac{4}{7}(v'\cdot v'')(v'\cdot g_0)(v\cdot g) + \frac{4}{7}(v'\cdot v'')(v'\cdot g)(v\cdot g_0) + \frac{4}{7}(v\cdot v')(v'\cdot g)(v''\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')(v'\cdot g_0)(v''\cdot g) + 4(v\cdot v'')(v\cdot b)(v\cdot g_0) + \frac{36}{7}(v\cdot v')(v'\cdot g')(v'\cdot g_0)$$

$$+\frac{4}{7}(v\cdot g)(v''\cdot g_0)v'^2 + \frac{9}{14}v'^2(v'\cdot g)(v'\cdot g_0)\Big\} - 64\frac{1}{(c^2-v^2)^6}\Big\{3(v\cdot v')^2(v\cdot g')(v\cdot g_0')$$

$$+2(v\cdot v')^2(v\cdot b)(v\cdot g_0) + 2(v\cdot v')(v\cdot v'')(v\cdot g')(v\cdot g_0) + \frac{1}{3}(v\cdot v'')^2(v\cdot g)(v\cdot g_0)$$

$$+\frac{18}{7}(v\cdot v')^2(v\cdot g')(v'\cdot g_0) + \frac{18}{7}(v\cdot v')^2(v\cdot g_0)(v'\cdot g') + \frac{4}{7}(v\cdot v')(v\cdot v'')(v\cdot g)(v'\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')(v\cdot g)(v\cdot g_0)(v'\cdot v'') + \frac{4}{7}(v\cdot v')(v\cdot v'')(v\cdot g_0)(v'\cdot g) + \frac{18}{7}v'^2(v\cdot v')(v\cdot g')(v\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')^2(v\cdot g)(v''\cdot g_0) + \frac{2}{7}v'^2(v\cdot v'')(v\cdot g)(v\cdot g_0) + \frac{9}{56}v'^4(v\cdot g)(v\cdot g_0)$$

$$+\frac{9}{14}(v\cdot v')^2(v'\cdot g)(v'\cdot g_0) + \frac{9}{7}v'^2(v\cdot v')(v\cdot g)(v'\cdot g_0) + \frac{192}{7}\{24(v\cdot v')^3(v\cdot g')(v\cdot g_0)$$

$$+8(v\cdot v')^2(v\cdot v'')(v\cdot g)(v\cdot g_0) + 9v'^2.(v\cdot v')^2(v\cdot g)(v\cdot g_0) + 12(v\cdot v')^3(v\cdot g)(v'\cdot g_0)\}$$

$$-576\frac{1}{(c^2-v^2)^8}(v\cdot v')^4(v\cdot g)(v\cdot g_0) - i\pi c^2\Big[\frac{4}{15}\frac{5c^2+v^2}{(c^2-v^2)^4}\{5[v\cdot(b\times b_0)]$$

$$+6[v'\cdot(b\times g_0')] + 2[v''\cdot(b\times g_0)]\} + \frac{4}{5}\frac{7c^2+v^2}{(c^2-v^2)^5}\{4(v\cdot v'')[v\cdot(b\times g_0)]$$

$$+ 12\,(v\cdot v')\,[v\cdot(b\times g_0')] + 6\,(v\cdot v')\,[v'\cdot(g'\times g_0')] + 3v'^2\,[v\cdot(g'\times g_0')]$$
$$+ 4\,(v\cdot v')\,[v'\cdot(b\times g_0)] + 2v'^2\,[v\cdot(b\times g_0)] + 2\,(v\cdot v')\,[v''\cdot(g'\times g_0)]$$
$$+ 2\,(v\cdot v'')\,[v'\cdot(g'\times g_0)] + 2\,(v'\cdot v'')\,[v\cdot(g'\times g_0)] + \frac{2}{3}(v\cdot v'')\,[v''\cdot(g\times g_0)]$$
$$+ \frac{1}{3}\,v''^2\,[v\cdot(g\times g_0)] + \frac{18}{7}\,v'^2\,[v'\cdot(g'\times g_0)] + \frac{4}{7}\,(v'\cdot v'')\,[v'\cdot(g\times g_0)]$$
$$+ \frac{2}{y}\,v'^2[v''\cdot(g\times g_0)] + \frac{16}{5}\,\frac{9c^2+v^2}{(c^2-v^2)^6}\,\{6\,(v\cdot v')^2[v\cdot(g'\times g_0')] + 4\,(v\cdot v')^2[v\cdot(b\times g_0)]$$
$$+ 4\,(v\cdot v')\,(v\cdot v'')\,[v\cdot(g'\times g_0)] + \frac{2}{3}\,(v\cdot v'')^2[v\cdot(g\times g_0)] + \frac{9}{7}\,v'^2(v\cdot v')\,[v'\cdot(g\times g_0)]$$
$$+ \frac{9}{28}\,v'^4\,[v\cdot(g\times g_0)] + \frac{4}{7}\,(v\cdot v')^2\,[v''\cdot(g\times g_0)] + \frac{8}{7}\,(v\cdot v')\,(v\cdot v'')\,[v'\cdot(g\times g^0)]$$
$$+ \frac{8}{7}\,(v\cdot v')\,(v'\cdot v'')\,[v\cdot(g\times g_0)] + \frac{4}{7}\,v'^2\,(v\cdot v'')\,[v\cdot(g\times g_0)]$$
$$+ \frac{16}{7}\,\frac{11c^2+v^2}{(c^2-v^2)^7}\,\{24\,(v\cdot v')^3\,[v\cdot(g'\times g_0)] + 12\,(v\cdot v')^2\,(v\cdot v'')\,[v\cdot(g\times g_0)]$$
$$+ 6\,(v\cdot v')^3\,[v'\cdot(g\times g_0)] + 9\,v'^2\,(v\cdot v')^2\,[v\cdot(g\times g_0)]\,]\,\}$$
$$+ \frac{288}{7}\,\frac{13c^2+v^2}{(c^2-v^2)^8}\,(v\cdot v')^4[v\cdot(g\times g_0)].$$

3. In the special case of a magneton of charge $4\pi e$ and moment $4\pi k$ describing a circular orbit at a constant speed and in such a way that the axis of the magneton is always perpendicular to the plane of the orbit, we find that if k is constant the rate of radiation of energy is

$$\frac{8}{3}\,\pi c e^2\,\frac{v'^2}{(c^2-v^2)^2} + \frac{8}{3}\,\pi c^2 e k\,\frac{v'^3}{v(c^2-v^2)^3} + \frac{16}{15}\,\pi c k^2\,\frac{v'^4}{v^2}\,\frac{2c^2+3v^2}{2(c^2-v^2)^4}$$

If e, v and v' are given the minimum value of this positive quantity is found to be

$$\frac{2\pi}{3}\,\frac{ce^2v'^2}{(c^2-v^2)^2}\,\frac{c^2+4v^2}{2c^2+3v^2}$$

and is thus about $\frac{1}{8}$ of the rate of radiation from the electric charge alone. Some years ago Dr. W. F. G. Swann expressed to me the desirability of calculating the radiation from an electron which rotates about its axis like a planet while describing a circular orbit. If such an electron can be treated as a magneton to a first approximation the above result is applicable. The fact that the radiation is reduced by rotation may indicate that revolving electrons do rotate.

4. A similar calculation for the case in which the axis of the magneton is tangential to the path indicates that the rate of radiation is greater than that from the electric charge alone.

5. It should be noticed that if $v' = \dot{v}'' = g'' = 0$ the expression for I vanishes. Hence when an electromagnetic pole and an electromagnetic doublet move together with constant velocity along a rectilinear path, it is possible for the moment of the doublet to change at a constant rate without there being any radiation of energy.

[1] *Electromagnetic Radiation*, Cambr. Univ. Press, 1912: *London, Phil. Mag.* (Ser. 6), 36, 1918, 243.

ON THE DISTRIBUTION OF THE APHELIA OF THE SECONDARY BODIES OF THE SOLAR SYSTEM

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The finding of a dependence of orbital eccentricity upon the relative masses of the components led to an examination of the directions of the aphelia of the secondary bodies of the solar system, a resumé of which is the principal object of the present paper.

There is little to guide us as yet in the interpretation of such a dependence. The first explanation which suggests itself is that it is a residual effect of capture. In the comets we have a very exaggerated effect of such a dependence. Their eccentricities are essentially unity and whether they come from interstellar space or from the outer planetary regions they are, for the few journeys which they perform about the sun, captured bodies. This is with reference to the comets with very long period and those with sensibly parabolic orbits. The relations of the orbits of those with comparatively short periods to the outer (and larger) planets establishes the fact that such comets have been captured for the sun by these different planets. We have, therefore, in the solar system at least one class of bodies which has been captured. A study of the orbital characteristics of these in connection with the other secondary bodies (having the sun also for primary) should disclose any similarities which may be significant.

One fact which stands out prominently in connection with this dependence of orbital eccentricity upon relative mass appears to be significant. There are two possibilities aside from that of *no* preference